Benha University Faculty of Engineering – Shoubra Department of Industrial Engineering Duration : 2 hours		Final Exam Course: Mathematics 1 Code: EMP 101 Date: May 12 , 2018

The exam consists of one page | No. of questions: 4 | Answer **All** questions | Total Mark: 40

Question 1

(a)Find **y`** from the following : — 6

(i) $y = x^3 + 3^x + 3x$ (ii) $y = x.\ln(1+x^3)$ (iii) $y = \tan x - \log x$

(iv) $y = 2 + e^x \sin x$ (v) $y = \sin x + \sin^3 x$ (vi) $y = (x + \ln x)^8$

(b) Write the Maclurin's expansion of : $f(x) = 2x + \sin x$. — 3

(c) Determine the maximum and minimum points of : $f(x) = 6x^2 - x^3$. — 2

(d)Find the limits: (i) $\lim\limits_{x\to 0} \dfrac{x - \sin x}{x^3 + x^2}$ (ii) $\lim\limits_{x\to 1} \dfrac{\ln x}{x - 1}$ (iii) $\lim\limits_{x\to\infty} \dfrac{2 + x^2}{x + 3x^2}$ — 3

Question 2

Find the integrals: — 6

(a) $\int (x^3 + e^x)\, dx$ (b) $\int (x^2 + 3)^2\, dx$ (c) $\int (x + \cos x)\, dx$

(d) $\int (3 + \sin 2x)\, dx$ (e) $\int (\frac{1}{x^3} - \sqrt{x})\, dx$ (f) $\int (3^x + 2^x)^2\, dx$

Question 3

(a) If $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 2 & 0 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$ — 8

Find, if possible, A + B, A + C, A.B , A.C, $C.A^t$, |A|, |C|.

(b) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$. — 4

Question 4

(a)Solve the linear systems : — 4

(i) $3x - 2y + z = 1,\ -x + y - 3z = -3,\ 2x - y - 2z = 2$.

(ii) $x + y - z = 3,\ -x + y + 2z = 6,\ x + y - 2z = 0$.

(b)If $z_1 = -2 + 2i$, $z_2 = 1 + i$. Find $z_1 + z_2$, $z_1.z_2$, $(z_1)^7$, $\sqrt[5]{z_2}$. — 4

Good Luck *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)(i) $y` = 3x^2 + 3^x . \ln 3 + 3$ (ii) $y` = 1. \ln(1 + x^3) + x\frac{3x^2}{1+x^3}$

(iii) $y` = \sec^2 x - \frac{1}{\ln 10}\frac{1}{x}$ (iv) $y` = 0 + e^x . \sin x + e^x . \cos x$

(v) $y` = \cos x + 3\,(\sin x)^2 \cos x$ (vi) $y` = 8(x + \ln x)^7 .\, (1 + \frac{1}{x})$

--*6 Marks*

(b) Since $f(0) = 0 + \sin 0 = 0$

$$f`(x) = 2 + \cos x, \quad then \quad f`(0) = 1 + 2 = 3$$

$$f``(x) = 0 - \sin x, \quad then \quad f``(0) = \sin 0 = 0$$

$$f```(x) = -\cos x, \quad then \quad f```(0) = -\cos 0 = -1$$

$$f^{(4)}(x) = \sin x, \quad then \quad f^{(4)}(0) = \sin 0 = 0$$

$$f^{(5)}(x) = \cos x, \quad then \quad f^{(5)}(0) = \cos 0 = 1$$

Then $f(x) = 0 + 3x + 0 + \frac{-1}{3!}x^3 + 0 + \frac{1}{5!}x^5$ ...

--*3 Marks*

(c) Since $f`(x) = 12x - 3x^2 = 0$. Then x = 0, x = 4.

From $f``(x) = 12 - 6x.$

We see that $f``(0) = 12 - 0 = 12,$ then x = 0 is minimum.

We see that $f``(4) = 12 - 24 = -12,$ then x = 4 is maximum.

--*2 Marks*

(d)(i) $\lim\limits_{x\to 0} \frac{x - \sin x}{x^3 + x^2} = \frac{0}{0} = \lim\limits_{x\to 0} \frac{1 - \cos x}{3x^2 + 2x} = \frac{0}{0} = \lim\limits_{x\to 0} \frac{\sin x}{6x + 2} = \frac{0}{2} = 0$

(ii) $\lim\limits_{x\to 1} \frac{\ln x}{x - 1} = \frac{0}{0} = \lim\limits_{x\to 1} \frac{\frac{1}{x}}{1} = 1$

(iii) $\lim\limits_{x\to \infty} \frac{2 + x^2}{x + 3x^2} = \frac{\infty}{\infty} = \lim\limits_{x\to \infty} \frac{2x}{1 + 6x} = \frac{2}{6} = \frac{1}{3}$

--*3 Marks*

Answer of Question 2

(a) $I = \frac{1}{4}x^4 + e^x + c$

(b) $I = \int(x^4 + 6x^2 + 9)\,dx = \frac{1}{5}x^5 + 2x^3 + 9x + c$

(c) $I = \frac{1}{2}x^2 + \sin x + c$

(d) $I = 3x - \frac{1}{2}\cos 2x + c$

(e) $I = \frac{x^{-2}}{-2} + \frac{2}{3}x^{3/2} + c$

(f) $I = \int(9^x + 4^x + 2\ 6^x)\,dx = \frac{1}{\ln 9}9^x + \frac{1}{\ln 4}4^x + \frac{2}{\ln 6}\ 6^x + c$

---*6 Marks*

Answer of Question 3

(a) A + C, A.B, |A| are impossible.

$$A + B = \begin{bmatrix} 4 & -1 \\ 4 & 1 \\ 3 & 3 \end{bmatrix}, \quad A.C = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 0 & 2 \end{bmatrix} . \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 3 \\ 8 & 2 \\ 8 & 0 \end{bmatrix},$$

$$C.A` = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 2 \\ 12 & 8 & 0 \end{bmatrix}$$

$|C| = 0 - 4 = -4$

---*8 Marks*

(b) $\begin{vmatrix} 2-\lambda & 3 \\ 1 & 0-\lambda \end{vmatrix} = (2-\lambda)(-\lambda) - 3 = \lambda^2 - 2\lambda - 3 = 0$. Then $\lambda_1 = -1$, $\lambda_2 = 3$

From the equation, $\begin{bmatrix} 2-\lambda & 3 \\ 1 & -\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = -1$, $\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then 3x + 3y = 0, x + y = 0

Then x = – y = any number except 0

Put y = 1, we get x = – 1 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For : $\lambda_2 = 3$, $\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then – x + 3y = 0, x – 3y = 0

Then x = 3y = any number except 0

Put y = 1, we get x = 3 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

---*4 Marks*

Answer of Question 4

(a)(i) $G = \left[\begin{array}{ccc:c} -1 & 1 & -3 & -3 \\ 3 & -2 & 1 & 1 \\ 2 & -1 & -2 & 2 \end{array}\right] \sim \left[\begin{array}{ccc:c} -1 & 1 & 1 & -3 \\ 0 & 1 & -8 & -8 \\ 0 & 1 & -8 & -6 \end{array}\right] \sim \left[\begin{array}{ccc:c} 1 & 1 & -3 & -3 \\ 0 & 1 & -8 & -8 \\ 0 & 0 & 0 & 2 \end{array}\right]$

Rank A = 2 and Rank G = 3. No solution.

(b)(ii)By regular Calculator, the solution is : (3, 3, 3).

--*4 Marks*

(b) $z_1 + z_2 = -1 + 3i, \quad z_1.z_2 = -4$

Since $z_1 = -2 + 2i = \sqrt{8}\, e^{\frac{3\pi}{4}i}$. Then $(z_1)^7 = 8^{\frac{7}{2}}\, e^{\frac{21\pi}{4}i}$

Since $z_2 = 1 + i = \sqrt{2}\, e^{\frac{\pi}{4}i}$. Then $\sqrt[5]{z_2} = \sqrt[10]{2}\, e^{\frac{\pi}{20}i}$

--*4 Marks*

Dr. Mohamed Eid